A Risk Primer: Return Variability and Risk

Or the Difference between Arithmetic and Geometric Rates of Return

One of the somewhat technical issues investors need to understand is the difference between arithmetic and geometric rates of return, sometimes referred to as average and compound return rates. An investor needs to understand the difference between the two not just to understand capital market reports that show both numbers. An investor needs to understand the difference in order to understand investment risk, for the difference between arithmetic and geometric rates of return lies at the root of investment risk.

The difference between arithmetic and geometric rates of return can be easily shown by example. Suppose you invest $1,000 in a security or market that produces a return of +50% in the first year and –50% in the second year. What's your average rate of return? A simple average of the return rates for the two years is 0%—this is the arithmetic average.

But if your average return for the two years is 0%, you should have $1,000 at the end of the two years, which you do not have. At the end of the first year, you have $1,500; and at the end of the second year, you have $750 – just as if you had lost 13.4% per year for each year, which is, in fact, the geometric (or compound) rate of return for the two years.

If you lose 50% in the first year and make 50% in the second year, you end up in the same place – with $500 at the end of the first year and $750 at the end of the second year. It makes no difference in what order the gains and losses occur. Your compound return rate is -13.4%.

The difference between the arithmetic or simple average and the geometric or compound average is not just a mathematical curiosity about past returns. The difference tells us something about the relative risk. Consider the following more robust example. Suppose we have three investments to choose from with the historical performances as shown in Figure RP.1.

Year	Hivee	Lovee	Novee
1	+1%	+6%	+1%
2	+51%	+1%	+1%
3	+11%	-4%	+1%
4	-49%	+1%	+1%
5	+21%	+6%	+1%
6	+1%	+1%	+1%
7	+1%	-4%	+1%
8	-9%	+1%	+1%
9	+1%	+6%	+1%
10	-19%	-4%	+1%
Arithmetic average return	+1.0%	+1.0%	+1.0%
Geometric average return	-2.3%	+0.9%	+1.0%
Ending value of $1,000 initial investment	$793	$1,096	$1,105

Figure RP.1: Historical Returns for Hypothetical Investments
Source: *Asset Allocation Advisor* calculations

The average arithmetic return for each investment is 1.0%. However, an investment in Novee would have increased by more than 10% over the ten years, while an investment in Hivee would

have lost more than 20% in ten years as reflected in its ending value and its negative geometric rate of return.

Although Hivee, Lovee, and Novee have the same arithmetic average rate of return, Hivee has the lowest compound rate of return because its returns are the most variable. In the case of Novee, which has a constant rate of return, its geometric rate of return is identical to its arithmetic rate of return (the only case in which this is true).

What should we expect from an investment in our three alternatives? We should expect a return of 1% per year from each. We can only discriminate among them once we know the variability of their returns, for it is the variability of an asset's return that causes its compound rate of return to fall below its average rate of return. The more variable the returns, the greater the discount from the arithmetic average to the geometric average. Figure RP.2 shows the impact of variability on returns. An asset with an arithmetic average return of 10% produces a geometric return of 9.5% at a standard deviation of 10%, but only 8.1% at a standard deviation of 20%. At a 30% standard deviation, its geometric return of 5.4% is not much more than half the arithmetic return.

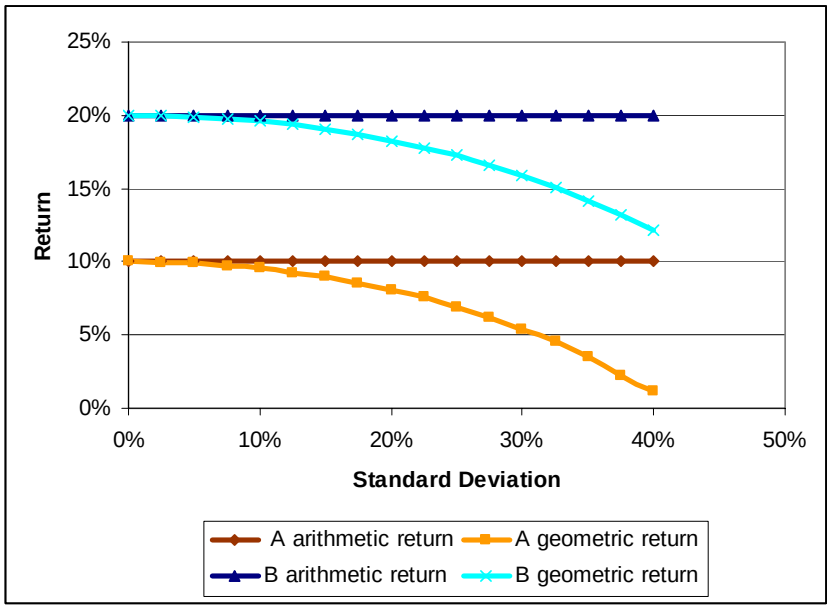

Figure RP.2: the Impact of Variability of Returns on an Investment's Geometric or Compound Rate of Return
Source: *Advisor* calculations

Variability of returns is at the heart of investment risk. It reduces long-term compound returns below the arithmetic average and thereby diminishes returns. And it exposes the investor to the possibility of loss.

The possibility of loss is actually a function of two factors, the variability of returns and the average arithmetic return. If the average return is high enough and the variability of returns is low enough, the possibility of loss is highly remote. It's only when the variability is high relative to the average return that the possibility of loss becomes

meaningful. Figure RP.3 illustrates this by showing the distribution of returns around the average for three different assets, all of which have an average return of 10% but with different variabilities. Returns on Asset A are not highly variable. Accordingly, returns are tightly grouped around the 10% average with only a small portion of the distribution of returns to the left of the 0% return point. Returns on Asset B are twice as variable as those on Asset A, and returns on Asset C are twice as variable as those on Asset B. Accordingly, returns on Assets B and C are more spread out and result in larger portions of the distributions falling to the left of the 0% return point, or in the loss category.

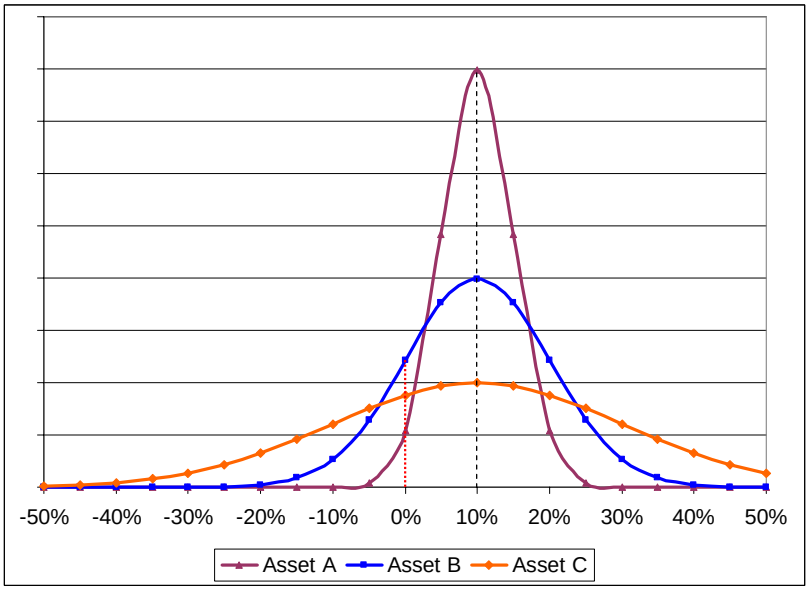

Figure RP.3: Comparisons of the Distributions of Returns on Assets with Identical Average Returns but Different Degrees of Variability among Returns.
Source: *Advisor* calculations

The last things investors need to understand about variable returns is that although it is mathematically possible to have an asset category with a high average return and a low variability of returns, **no such thing exists in reality**. We never encounter assets like those pictured above with identical average returns but different variabilities. Why not? Because if they existed, they would violate the "no free lunch" principle. Investors in Asset A realize a higher compound rate of return with a lower probability of loss than investors in Assets B or C. More return with less risk is a free lunch. If such an asset existed, investors would flock to buy Asset A. They would bid up the price of Asset A, and thereby reduce its average return and eliminate the starting parity between Asset A and Assets B and C. The market does not allow free lunches. An asset with a high rate of return invariably comes with a high variability of returns. Risk and reward come packaged together and cannot be separated.

The variability of returns is at the root of investment risk. It is the reason for the difference between the simple average of past returns, the arithmetic average, and the long-term compound rate of return, the geometric average, which an investor will realize over time. And it is the reason for the risk of loss that is implicit in higher returning assets.